

Tuning the Cosmological Constant, Broken Scale Invariance, Unitarity

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Abstract

We study gravity coupled to a cosmological constant and a scale but not conformally invariant sector. In Minkowski vacuum, scale invariance is spontaneously broken. We consider small fluctuations around the Minkowski vacuum. At the linearised level we find that the trace of metric perturbations receives a positive or negative mass squared contribution. However, only for the Fierz-Pauli combination the theory is free of ghosts. The mass term for the trace of metric perturbations can be cancelled by explicitly breaking scale invariance. This reintroduces fine-tuning. Models based on four form field strength show similarities with explicit scale symmetry breaking due to quantisation conditions.

1 Introduction

The cosmological constant problem can be looked at from various angles, a classic and some more recent reviews are listed in [1–7]. It is essentially the fact that known contributions to the cosmological constant are many orders of magnitude above its observed value. In the present note, we will discuss the question whether it is possible to design a sector with an adjustable contribution to the cosmological constant. We impose such a sector to be scale invariant. The different classical solutions are related by scale transformations. Choosing one solution amounts to tuning the cosmological constant to a given value. On the other hand, this tuning sector should not be conformally invariant because a traceless energy momentum tensor cannot change an effective cosmological constant. There is a theorem that such theories are not unitary [8–11]. However, we also need to mimic a cosmological constant implying that the energy momentum tensor of the tuning sector is covariantly constant. We can achieve that with a somewhat pathological setup for which we are not sure whether the theorem applies. For instance, perturbations around a classical solution can be gauged away leaving only gravity at the linearised level.

First, we consider a rather contrived model capable of contributing an arbitrary cosmological constant at the classical level. We also investigate perturbations of these classical solutions and impose consistency conditions. In particular, gravity should be quantisable as an effective theory. This is necessary to maintain a quantum picture for mass attraction due to the exchange of gravitons. We find a mass term just for the trace of metric perturbations. In such a theory there are negative norm states and the mass term has to be cancelled. This can be done by explicitly breaking scale invariance. Then the observed value of the cosmological constant is fixed in terms of its bare value and model parameters, the tuning feature is lost.

We suspect that the appearance of a mass term just for the trace of metric perturbations is generically related to spontaneously broken scale invariance. As another example we consider a tuning sector made of four form field strength. Indeed, the same problem arises. Here, however, scale invariance is broken by the quantisation condition leaving the cosmological constant tunable by discrete amounts which is crucial for the string landscape approach to the cosmological constant problem.

2 Minkowski Vacua

2.1 Classical Solution

Our starting point is the following action

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{2\kappa^2} - \Lambda - \lambda \left| \frac{\gamma}{g} \right|^\alpha \right\}, \quad (1)$$

where g denotes the determinant of the metric tensor $g_{\mu\nu}$, R is the scalar curvature and the cosmological constant Λ is understood to include a bare contribution together with all

quantum corrections. The tuning sector consists of four scalars ϕ^M , $M \in \{0, 1, 2, 3\}$ which are combined into a four by four matrix

$$\gamma_{\mu\nu} = \partial_\mu \phi^M \partial_\nu \phi^N \eta_{MN}. \quad (2)$$

We are using the mostly plus convention in which the Minkowski metric $\eta_{MN} = \text{diag}(-1, 1, 1, 1)$. The last term in (1) will be our tuning sector. It is invariant under diffeomorphisms and scale transformation (to be discussed further in section 2.2). The scalars ϕ^M could be viewed as a coordinate of a space filling brane wrapping a parallel universe. The metric induced on the worldvolume of that brane is $\gamma_{\mu\nu}$. Parallel universes in context of the cosmological constant have been considered e.g. in [12] to improve an old proposal by Tseytlin [13]. In our case the parallel universe is just, barring the space filling brane, empty Minkowski space. There are no gravitational interactions in the parallel universe. The Lorentz isometry of the parallel universe appears as global symmetry in our universe. In a theory with quantised gravity global symmetries are believed to be absent, see e.g. [14]. So, finally the Lorentz isometry should be gauged. For now, we are not quantising gravity and neglect corresponding effects. The last term in (1) is some weighted geometric mean between the squareroots of the determinants of the induced metric on the brane and our spacetime metric. For $\alpha = 1/4$ we have the usual geometric mean, for $\alpha = 0$ just another cosmological constant, and for $\alpha = 1/2$ our space filling brane completely decouples from gravity. At the moment, the only motivation for the last term in (1) is that it seems to help with the fine tuning problem of the cosmological constant.

Variation with respect to the metric yields Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa^2 \left(\Lambda + \lambda (1 - 2\alpha) \left| \frac{\gamma}{g} \right|^\alpha \right) g_{\mu\nu}. \quad (3)$$

The field equations for the scalars ϕ^M read

$$\partial_\mu \left((\sqrt{-g})^{1-2\alpha} (-\gamma)^\alpha (\gamma^{-1})^{\mu\nu} \partial_\nu \phi^M \right) = 0 \quad (4)$$

where $(\gamma^{-1})^{\mu\nu}$ is just the $\mu\nu$ component of the inverse of $\gamma_{\mu\nu}$. i.e. the metric is not involved in raising the indices. A very simple solution is

$$g_{\mu\nu} = \eta_{\mu\nu} \text{ and } \partial_\mu \phi^M = C \delta_\mu^M, \quad (5)$$

where the integration constant C is fixed by (3)

$$\Lambda = \lambda (2\alpha - 1) |C|^{8\alpha}. \quad (6)$$

So, any cosmological constant Λ whose sign matches the sign of the RHS of (6) can be sequestered from gravity in that it does not result in spacetime curvature. Notice that (5) relates the tuning sector to the vierbein of Minkowski spacetime. This bears some similarity to the ‘‘vacuum variable’’ of [15]. Notice however, that the tuning (6) does not solve the fine tuning problem of the cosmological constant by itself. In the context of unimodular

gravity [16,17] this is discussed in [2]. Unimodular gravity is just a gauge fixed version of Einstein gravity. Unimodularity can be imposed with a Lagrange multiplier. The Bianchi identity forces the Lagrange multiplier to take the role of a cosmological constant which now appears as an integration constant. Our setup is different in that we have added a scale invariant sector. Still, as we will see later, it does not provide a solution to the fine tuning problem of the cosmological constant.

2.2 Scale Invariance without Conformal Invariance

Conformal symmetry and its relation to the cosmological constant has been already addressed in [1] and references therein. A more recent discussion concerning a proposal of Kaloper and Padilla [18] can be found in [19]. Here, we will argue on a classical level that the tuning sector in (1) is scale invariant but not conformally invariant. Let us specify scaling transformations in our action (1). We focus on the last term and, for simplicity, take $g_{\mu\nu} = \eta_{\mu\nu}$, i.e. we consider the action

$$S_\gamma = -\lambda \int d^4x |\gamma|^\alpha. \quad (7)$$

This action is invariant under global scale transformations

$$x^\mu \longrightarrow \Omega x^\mu, \quad (8)$$

provided the four scalars transform as

$$\phi^M(x) = \Omega^\Delta \tilde{\phi}^M(\Omega x) \quad \text{with} \quad \Delta = \frac{1}{2\alpha} - 1. \quad (9)$$

The corresponding Noether current is

$$D_\mu = x^\rho T_{\mu\rho} - J_\mu. \quad (10)$$

Here, $T_{\mu\nu}$ is the energy momentum tensor which can be either obtained by taking the variational derivative with respect to the metric $g_{\mu\nu}$ before restricting to the Minkowski metric or by computing the Noether current belonging to constant shift symmetry of the coordinates x^μ in (7). Either way it is (cf (3))

$$T_{\mu\nu} = \lambda (1 - 2\alpha) |\gamma|^\alpha \eta_{\mu\nu}. \quad (11)$$

The second term in (10) is called virial current [20]. It reflects the non vanishing scaling dimension of our fields and is given by

$$J^\mu = \lambda (1 - 2\alpha) |\gamma|^\alpha (\gamma^{-1})^{\mu\nu} \phi^M \partial_\nu \phi^N \eta_{MN}. \quad (12)$$

Using equations of motion one can check that the trace of the energy momentum tensor is indeed given by the divergence of the virial current,

$$T^\mu_\mu = \partial_\mu J^\mu = 4\lambda (1 - 2\alpha) |\gamma|^\alpha. \quad (13)$$

The non vanishing trace is a signal that scale symmetry is strictly global and not enhanced to conformal symmetry. This is essential, because otherwise the tuning sector could not cancel the cosmological constant. There could, however, be an improved energy momentum tensor whose trace vanishes. Indeed, our trace can be expressed as¹

$$T_\mu^\mu = \partial^\kappa \partial^\lambda L_{\kappa\lambda} = \square L, \quad \text{with } L_{\kappa\lambda} = \eta_{\kappa\lambda} L, \quad L = \frac{\lambda}{2} (1 - 2\alpha) |\gamma|^\alpha x^2, \quad (14)$$

where we used that (4) implies constant γ for $\alpha \neq \frac{1}{2}$. (This can be seen by multiplying (4) with $\partial_\lambda \phi_M$, summing over M , using Leibniz rule and $\partial_\mu \gamma = \gamma (\gamma^{-1})^{\rho\kappa} \partial_\mu \gamma_{\rho\kappa}$.) Then there is a traceless, conserved improved tensor [20]

$$T_{\mu\nu} + \frac{1}{2} (\partial_\mu \partial_\rho L^\rho_\nu + \partial_\nu \partial_\rho L^\rho_\mu - \partial^2 L_{\mu\nu} - \eta_{\mu\nu} \partial_\rho \partial_\kappa L^{\rho\kappa}) + \frac{1}{6} (\eta_{\mu\nu} \partial^2 L^\rho_\rho - \partial_\mu \partial_\nu L^\rho_\rho), \quad (15)$$

which in our case, however, vanishes. A similar discussion applies to models involving a four form field strength [21–24]. For other, related models see e.g. [25–27]. Having a theory which is scale but not conformally invariant indicates potential problems². Indeed there is a theorem stating that, provided Poincaré invariance holds, such a theory is not unitary [8–11]). However, our tuning sector is a pathological theory and we are a priori not sure whether the theorem applies. We have seen that the trace of the energy momentum tensor can be expressed in terms of a D’Alambertian. Often this would imply conformal invariance whereas our Lagrangian is not invariant under local scale transformations. We will see in the next subsection that all perturbations can be gauged away and the resulting linearised gravity is not unitary. Hence, on that note, the theorem is confirmed.

2.3 Background Field Expansion and Ghosts

To investigate the question of unitarity we check whether our model suffers from ghosts, at a perturbative level. We will consider an action obtained by expanding (1) around the Minkowski solution till second order in the fluctuations. Non vanishing Λ is assumed. We perturb metric and scalars by small fluctuations $h_{\mu\nu}$ and ϵ^M

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \partial_\mu \phi^M = C (\delta_\mu^M + \partial_\mu \epsilon^M). \quad (16)$$

Expanding the Einstein-Hilbert term leads to the standard result

$$\int d^4x \sqrt{-g} R \approx \int d^4x \left\{ \frac{1}{4} \partial_\lambda h_\rho^\rho \partial^\lambda h_\kappa^\kappa - \frac{1}{4} \partial_\lambda h_\kappa^\rho \partial^\lambda h_\rho^\kappa + \frac{1}{2} \partial_\lambda h_\rho^\lambda \partial^\kappa h_\kappa^\rho - \frac{1}{2} \partial_\kappa h_\lambda^\lambda \partial^\rho h_\rho^\kappa \right\}, \quad (17)$$

¹Notice, however, $J_\mu \neq \partial_\mu L = D_\mu + J_\mu$, i.e. with our L one cannot construct a conserved conformal current in contrast to the L discussed e.g. in [10].

²For $\alpha = 1/2$, action (7) is invariant under general coordinate transformations, i.e. in particular under conformal transformations. An interesting limit may be given by $\alpha \rightarrow \frac{1}{2}$, $\lambda \rightarrow \infty$ such that $(\alpha - 1/2)\lambda$ remains finite. Eq. (3) shows that there is still a non vanishing contribution to the cosmological constant. On the other hand, as long as α differs only very little from $1/2$ conformal symmetry is broken and the non vanishing trace in (13) suggests that this is also true in the limit.

where indices are raised and lowered with the Minkowski metric. For us more interesting is the expansion of the remaining two terms in (1) resulting in

$$\int d^4x \sqrt{|g|} \left\{ \Lambda + \lambda \left| \frac{\gamma}{g} \right|^\alpha \right\} \approx -\frac{\Lambda}{1-2\alpha} \int d^4x \left\{ 2\alpha + \frac{1}{2}\alpha \left(\alpha - \frac{1}{2} \right) (2\partial_\mu \epsilon^\mu - h^\lambda_\lambda)^2 \right\}, \quad (18)$$

where (6) has been employed. The first contribution is an irrelevant constant which does not couple to the metric. At first, it seems that as long as the coefficient in front of the second term is positive we have a stable solution. That is, the Euclidean version of the action is positive definite or, phrasing it as the authors of [15], we have positive vacuum compressibility. The condition is explicitly

$$\alpha\Lambda > 0. \quad (19)$$

So, even if we were allowed to introduce Lagrangians like (1) by hand we would have to decide on the sign of the cosmological constant we want to cancel.

The situation is actually worse. Our spacetime is not Euclidean and we have to worry about negative norm states. In our model, two Lorentz groups appear. There is the global Lorentz symmetry rotating the four scalars ϕ^M and then there is the isometry of our solution. Our background breaks the two Lorentz groups into a diagonal one. The scalars ϕ^M ‘metamorphose’ into four-vectors ϵ^μ . As is well known from quantum electrodynamics one needs gauge symmetry to decouple ghosts from the space of physical states. Here, we do not have the $U(1)$ of electrodynamics but we do have spacetime diffeomorphisms

$$x^\mu \rightarrow x^\mu + \xi^\mu(x). \quad (20)$$

Viewing ξ as a quantity of the same order as our perturbations (16) the leading order transformations are

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad \epsilon_\mu \rightarrow \epsilon_\mu + \xi_\mu, \quad (21)$$

and we recognize a gauge invariant combination in (18). Now, we could impose a (partial) gauge fixing condition

$$\partial_\mu \epsilon^\mu = 0. \quad (22)$$

We end up with a theory of massive gravity with a mass term only for the trace of metric perturbations. Probably this is related to spontaneous breaking of scale invariance. It is proven in [28] that, at the linearised level³, the only consistent pure gravity theories are either the Einstein Lagrangian (17) or the Fierz-Pauli Lagrangian. The mass term in that Lagrangian contains the combination $h_{\mu\nu} h^{\mu\nu} - (h^\lambda_\lambda)^2$ which does not appear in our expansion even for particular values of parameters. So, the only way our model can be consistent is to replace the inequality (19) by an equality. But then the model is useless as far as the fine tuning problem for the cosmological constant is concerned.

³Here, ‘linearised’ refers to the equations of motion and corresponds to an action quadratic in perturbations. The remaining orders in perturbation theory are discussed in [29,30]. For reviews see e.g. [31,32].

2.4 Explicitly Breaking Scale Invariance

We break scaling invariance by supplementing (1) with another contribution

$$S = \int d^4x \sqrt{-g} \left\{ \frac{R}{2\kappa^2} - \Lambda - \lambda_1 \left| \frac{\gamma}{g} \right|^\alpha - \lambda_2 \left| \frac{\gamma}{g} \right|^\beta \right\}, \quad (23)$$

with $\beta \neq \alpha$. Assigning our previous scaling dimension (9) to ϕ^M we see that the last term breaks scaling invariance explicitly. (Actually, viewing λ_2 as an expectation value of a field with dimension $\Delta_{\lambda_2} = 4(1 - \beta/\alpha)$, the action (23) could still be traced back to spontaneous breakdown of scale invariance.) Again, we make the ansatz

$$g_{\mu\nu} = \eta_{\mu\nu} \quad , \quad \partial_\mu \phi^M = C \delta_\mu^M. \quad (24)$$

Einstein's equations are solved if C is chosen such that

$$0 = \Lambda + \lambda_1 (1 - 2\alpha) |C|^{8\alpha} + \lambda_2 (1 - 2\beta) |C|^{8\beta}. \quad (25)$$

In the linearised theory (18) is replaced by

$$\begin{aligned} \int d^4x \sqrt{|g|} \left\{ \Lambda + \lambda_1 \left| \frac{\gamma}{g} \right|^\alpha + \lambda_2 \left| \frac{\gamma}{g} \right|^\beta \right\} \approx \int d^4x \left\{ 2\alpha\lambda_1 |C|^{8\alpha} + 2\beta\lambda_2 |C|^{8\beta} + \right. \\ \left. - \frac{1}{4} \left(\lambda_1 \alpha (1 - 2\alpha) |C|^{8\alpha} + \lambda_2 \beta (1 - 2\beta) |C|^{8\beta} \right) (h_\lambda^\lambda)^2 \right\}, \end{aligned} \quad (26)$$

where the gauge fixing condition (22) has been imposed. Still, we do not have a chance to obtain the Fierz-Pauli action. But we can obtain linearised Einstein by choosing

$$\lambda_2 = -\lambda_1 \frac{\alpha (2\alpha - 1)}{\beta (2\beta - 1)} |C|^{8(\alpha - \beta)}. \quad (27)$$

Equations (25) and (27) are two conditions for one integration constant C leaving us with one fine-tuning condition. We could get another integration constant by introducing a second set of scalars $\tilde{\phi}^M$, but this would leave us with a massless four-vector in the linearised theory. We can set only one four-vector to zero by gauge fixing and the remaining one would miss the usual $U(1)$ gauge symmetry.

An alternative way to explicitly break scale invariance of the tuning sector is to introduce a cutoff M which should be related to Λ as

$$\Lambda \sim M^4. \quad (28)$$

Now, one could wonder whether there is a parameter region for which the mass of the ghost is above the cutoff scale. If so, unitarity would be effectively restored. We estimate the mass of the ghost in appendix A as

$$m_g^2 \sim \frac{\alpha \Lambda}{M_{\text{Planck}}^2} \sim \alpha \frac{M^2}{M_{\text{Planck}}^2} M^2. \quad (29)$$

So, if we chose

$$|\alpha| \gg \frac{M_{\text{Planck}}^2}{M^2} \quad (30)$$

and the sign such that (19) holds we could decouple the ghost. However, at the same time, we would also decouple h_μ^μ . Diffeomorphism invariance would be partially broken and effectively we would have unimodular gravity. As already mentioned, this is a gauge fixed version of Einstein gravity for which the usual bare cosmological constant appears as an integration constant.

3 Curved Vacua

In this section we argue that a mismatch of the fine tuning condition derived in the previous section leads to spacetime curvature.

3.1 De Sitter Space: Classical Solution

Using conformal time coordinates the de Sitter metric reads

$$ds^2 = \frac{\rho^2}{\tau^2} (-d\tau^2 + \delta_{ij} dx^i dx^j), \quad (31)$$

where $i, j \in \{1, 2, 3\}$ label spatial directions. The quantity ρ is related to the observed cosmological constant Λ_{obs}

$$\rho^2 = \frac{3}{\kappa^2 \Lambda_{\text{obs}}}. \quad (32)$$

Here, Λ_{obs} is the cosmological constant one would deduce by measuring spacetime curvature and assuming a cosmological constant as its only source. Now, relating $\partial_\mu \phi^M$ to the vierbein of the metric (31) does not solve the ϕ^M equations (4) since de Sitter space has a spin connection which cannot identically vanish. The solution is to choose a vierbein of a metric with the same determinant, i.e.⁴

$$\partial_\mu \phi^M = C \begin{cases} \frac{\rho^4}{\tau^4} & \text{for } \mu = M = 0, \\ \delta_j^i & \text{for } M = i, \mu = j, \\ 0 & \text{else.} \end{cases} \quad (33)$$

Plugging this into the Einstein equation (3) we find the observed cosmological constant to be determined by

$$\Lambda_{\text{obs}} = \Lambda + \lambda (1 - 2\alpha) |C|^{8\alpha}. \quad (34)$$

So, at this stage, it seems that by adjusting the integration constant C we can get any value for the observed cosmological constant.

⁴In the same way as the authors of [27] we could alternatively modify our action such that it is invariant under local Lorentz rotations of the ϕ^M . Here, we are not pursuing this possibility further.

3.2 Background Field Expansion

Here, we consider again small fluctuations around the classical solution

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \ , \ \partial_\mu \phi^M = \partial_\mu \bar{\phi}^M + C \partial_\mu \epsilon^M, \quad (35)$$

where barred quantities denote the classical solution (31) and (33). Here, it is useful to split the action into two terms, of which the first one gives the standard metric fluctuations in de Sitter space

$$\begin{aligned} \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} - \Lambda_{\text{obs}} \right) = & \frac{1}{2\kappa^2} \int d^4x \sqrt{-\bar{g}} \left\{ \frac{1}{4} \nabla_\lambda h_\rho^\rho \nabla^\lambda h_\kappa^\kappa - \frac{1}{4} \nabla_\lambda h_\kappa^\rho \nabla^\lambda h_\rho^\kappa \right. \\ & \left. + \frac{1}{2} \nabla_\lambda h_\rho^\lambda \nabla^\kappa h_\kappa^\rho - \frac{1}{2} \nabla_\kappa h_\lambda^\lambda \nabla^\rho h_\rho^\kappa + \frac{\Lambda_{\text{obs}}}{2} \left(h^{\kappa\lambda} h_{\kappa\lambda} - \frac{1}{2} (h^\lambda_\lambda)^2 \right) \right\}, \end{aligned} \quad (36)$$

where indices are raised and lowered with $\bar{g}_{\mu\nu}$ and covariant derivatives, denoted by nabla, are defined in terms of Christoffel symbols computed from $\bar{g}_{\mu\nu}$. For the remaining contribution we find

$$\begin{aligned} \int d^4x \sqrt{-g} \left(\Lambda - \Lambda_{\text{obs}} + \lambda \left| \frac{\gamma}{g} \right|^\alpha \right) = & 2\alpha\lambda |C|^{8\alpha} \int d^4x \frac{\rho^4}{\tau^4} \left\{ 1 + \right. \\ & \left. + \left(\alpha - \frac{1}{2} \right) \left(\frac{\tau^4}{\rho^4} \partial_0 \epsilon^0 + \partial_i \epsilon^i - \frac{1}{2} h^\lambda_\lambda \right)^2 \right\}. \end{aligned} \quad (37)$$

The expression in the second line is again the combination invariant under diffeomorphisms. Indeed, with

$$\delta h_\mu^\mu = 2\nabla_\mu \xi^\mu \text{ and } C\delta\epsilon^M = \xi^\mu \partial_\mu \bar{\phi}^M, \quad (38)$$

one finds

$$\delta h_0^0 = 2\partial_0 \xi^0 - \frac{2}{\tau} \xi^0, \quad (39)$$

$$\delta h_i^i = 2\partial_i \xi^i - \frac{6}{\tau} \xi^0, \quad (40)$$

$$\partial_0 \delta \epsilon^0 = \frac{\rho^4}{\tau^4} \left(\partial_0 \xi^0 - \frac{4}{\tau} \xi^0 \right), \quad (41)$$

$$\partial_i \delta \epsilon^i = \partial_i \xi^i. \quad (42)$$

This symmetry can be used to remove the ϵ^M from (37) and we end up with a mass term for metric fluctuations which is not of the Fierz-Pauli form⁵. To cancel this mass term we consider the modification (23). Then (34) is replaced by

$$\Lambda_{\text{obs}} = \Lambda + \lambda_1 (1 - 2\alpha) |C|^{8\alpha} + \lambda_2 (1 - 2\beta) |C|^{8\beta}. \quad (43)$$

⁵Massive gravity in curved space is e.g. reviewed in section 5 of [31].

Imposing the last terms in (36) to be the only mass terms for metric perturbations yields again condition (27). From that the integration constant C is determined and (43) provides a prediction for the observed value of the cosmological constant in terms of model parameters. So far, our ansatz implied a positive constant on the LHS of (43). If parameters provide a negative prediction we have to replace de Sitter space by Anti de Sitter space which we briefly discuss in the next subsection.

3.3 Anti De Sitter Space

For anti de Sitter space the discussion is very similar. Now the metric reads

$$ds^2 = \frac{\rho^2}{z^2} (-dt^2 + dx^2 + dy^2 + dz^2), \quad (44)$$

where ρ^2 is related to the observed cosmological constant according to

$$\rho^2 = -\frac{3}{\kappa^2 \Lambda_{\text{obs}}}. \quad (45)$$

The solution for the scalars is

$$\partial_\mu \phi^M = C \begin{cases} \frac{\rho^4}{z^4} & \text{for } \mu = M = 3, \\ \delta_\mu^M & \text{for } M, \mu \in \{0, 1, 2\}, \\ 0 & \text{else.} \end{cases} \quad (46)$$

The rest of the discussion follows quite closely the de Sitter case with the conclusion that (27) has to be imposed. Then (43) serves as a prediction for the observed cosmological constant, in case the RHS of (43) is negative.

4 Relation to Four Form Field Strength Scenario

One of the, at least naively, simplest tuning mechanisms is based on adding a four form field strength [21–24]

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} - \Lambda - \frac{Z}{2 \cdot 4!} F_4^2 \right), \quad (47)$$

where $F_4 = dA_3$ is the field strength of a three form gauge potential and

$$F_4^2 = F_{\mu\nu\lambda\kappa} F^{\mu\nu\lambda\kappa} = 4! \frac{f^2}{g} \quad \text{with } f = F_{0123}. \quad (48)$$

Let us ignore first any quantisation condition on the four form which we will consider later. Then the F_4 part of our action is scale invariant without being conformally invariant. The equations of motion are

$$\nabla_\mu F^{\mu\nu\kappa\lambda} = 0, \quad (49)$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa^2 \left(\Lambda - Z \frac{f^2}{2g} \right) g_{\mu\nu}. \quad (50)$$

The solution to (49) is

$$f = C\sqrt{-g}, \quad (51)$$

with C being an integration constant. Plugging this into the Einstein equation (50) yields a maximally symmetric solution corresponding to

$$\Lambda_{\text{obs}} = \Lambda + \frac{ZC^2}{2}. \quad (52)$$

Now, we assume that C is chosen such that $\Lambda_{\text{obs}} = 0$, and consider perturbations

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad , \quad f = C(1 + \epsilon) \quad \text{with} \quad C\epsilon = \partial_{[0}\alpha_{123]}, \quad (53)$$

where α_3 is the perturbation of the three form gauge potential. The interesting part from the expansion is

$$\frac{Z}{2} \int d^4x \left(\sqrt{-g}C^2 + \frac{f^2}{\sqrt{-g}} \right) = \frac{ZC^2}{2} \int d^4x \left\{ 2 + \left(\epsilon - \frac{h_\lambda^\lambda}{2} \right)^2 \right\}. \quad (54)$$

The combination inside the last square is again gauge invariant. (F_{0123} transforms into F_{0123} times the determinant of the transformation matrix.) We can employ diffeomorphisms to gauge away ϵ and obtain once again a mass term just for the trace of the metric perturbation, and not for the Fierz-Pauli combination. Probably this is a quite generic feature of spontaneously broken scale invariance.

Now, scale invariance is broken explicitly by a quantisation condition on F_4 [24]

$$f = \frac{en}{Z}\sqrt{-g}. \quad (55)$$

Thereby, the three form gauge field ceases to be a dynamical field and it is natural to plug (55) into the action (47) yielding

$$\Lambda_{\text{obs}} = \Lambda - \frac{e^2 n^2}{2Z} \quad (56)$$

and perturbations just lead to linearised Einstein theory possibly in a curved background. However, we could also follow [33] and plug (55) into the equation of motion resulting in a sign difference

$$\Lambda_{\text{obs}} = \Lambda + \frac{e^2 n^2}{2Z}. \quad (57)$$

In this picture, the quantisation condition (55) relates gauge form and metric perturbations

$$2\epsilon = h_\lambda^\lambda. \quad (58)$$

The unwanted mass term in (54) drops out for this configuration. So, from either perspective, breaking scale invariance by the quantisation condition cancels the mass term for the trace of metric perturbations. The cosmological constant is predicted by the value of the four form. The great advantage of this scenario has been pointed out in [24]. String theory can provide many four form contributions. The observed value of the cosmological constant depends on the radius of a vector in a multi dimensional charge lattice. If the dimension is big enough it can actually cancel a Planck sized Λ with the required precision. For a recent philosophical discussion see [34].

5 Conclusions

The question whether it is possible to design a sector with an adjustable contribution to an effective cosmological constant received, once again, a negative answer. Still, we believe that our investigation led to some interesting theoretical insight. As a tuning sector, we envisaged a scale but not conformally invariant model. There is a theorem stating that such a theory is not unitary [8–11]. Indeed, if we decouple the tuning sector from everything else and expand around a classical solution we find massless vectors without gauge invariance. Such a theory is not unitary. However, the sector couples to gravity and, via gravity, to another non scale invariant contribution, the bare cosmological constant. Taking perturbations of the metric into account we find a diffeomorphism invariant action. Using that symmetry to gauge away the massless vectors yields massive gravity with the wrong mass term, however. The non unitarity has been ‘transferred’ to gravity. Our example indicates that the theorem about non unitarity of scale but not conformally invariant systems holds also when such systems are coupled via gravity to a scale invariance breaking sector. In our setup we can cancel the graviton mass term by explicitly breaking scale invariance and introducing fine-tuning. In models based on four form field strength similar considerations apply. Here, scale invariance is broken explicitly by a quantisation condition. This removes the graviton mass term from the perturbed action.

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A Estimating the Ghost Mass

In this appendix we give a rough estimation of the ghost mass for linearised gravity with a mass term being off the Fierz-Pauli tuning, that is we take the action

$$\int d^4x \left\{ M_{\text{Planck}}^2 \sqrt{-g} R - f^4 \left(a h_{\mu\nu} h^{\mu\nu} + b (h_\mu^\mu)^2 \right) \right\}, \quad (59)$$

where the first term is to be replaced by its linearised version (17). We consider dimensionless fields, M_{Planck} and f have mass dimension one. The Fierz-Pauli tuning corresponds to $a + b = 0$. For the case of small non zero $a + b$ the mass of the ghost has been estimated in [35]. In the following we extend the argument for finite $a + b$. First, we leave the so called unitary gauge and restore diffeomorphism invariance by replacing

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + 2\partial_{(\mu}\chi_{\nu)}, \quad (60)$$

where χ_μ transforms as vector and has mass dimension -1 . The ghost comes from the longitudinal part of χ . Since we are just interested in the ghost we consider

$$\chi_\mu = \frac{1}{f^2} \partial_\mu \pi, \quad (61)$$

with π being a scalar of mass dimension zero. The mass term will lead to mixed terms

$$-4f^2 (ah^{\mu\nu} \partial_\mu \partial_\nu \pi + bh_\mu^\mu \square \pi). \quad (62)$$

There is also a term quadratic in π containing four derivatives. This term signals the appearance of a ghost. For the Fierz-Pauli tuning ($b = -a$) the term quadratic in π vanishes and the mixing of h and π in (62) can be cancelled by a Weyl transformation

$$h_{\mu\nu} = \hat{h}_{\mu\nu} + 4 \frac{af^2}{M_{\text{Planck}}^2} \eta_{\mu\nu} \pi. \quad (63)$$

After this redefinition the action reads ($\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \hat{h}_{\mu\nu}$)

$$\begin{aligned} \int d^4x \left\{ M_{\text{Planck}}^2 \sqrt{-\hat{g}} \hat{R} - f^4 \left(a \hat{h}^{\mu\nu} \hat{h}_{\mu\nu} + b \left(\hat{h}_\mu^\mu \right)^2 \right) \right. \\ - \frac{8af^4}{M_{\text{Planck}}^2} (5a + 8b) \pi \square \pi - 4f^2 (a + b) \hat{h}_\mu^\mu \square \pi - 4(a + b) \square \pi \square \pi \\ \left. - \frac{8af^6}{M_{\text{Planck}}^2} (a + 4b) \hat{h}_\mu^\mu \pi - \frac{64a^2 f^8}{M_{\text{Planck}}^4} (a + 4b) \pi^2 \right\} \end{aligned} \quad (64)$$

Following [35], we want to identify the ghost mass as the momentum scale at which the kinetic energy changes sign. For this reason, we ignore the mass terms in the last line of (64). In the near Fierz-Pauli limit ($a + b \sim 0$) the coupling between π and metric perturbations can be neglected and therefore the authors of [35] focused just on the π sector (first and last term in second line of (64)). We are ultimately interested in the $a = 0$ case for which near Fierz-Pauli and massless limit coincide. Therefore, we will consider finite $a + b$. It is useful to reorganise metric perturbations as follows. First, we decompose them into traceless ($\bar{h}_{\mu\nu}$) and trace ($\hat{h} = \hat{h}_\rho^\rho$) part

$$\hat{h}_{\mu\nu} = \bar{h}_{\mu\nu} + \frac{1}{4} \eta_{\mu\nu} \hat{h}. \quad (65)$$

Further we transform our expressions to momentum space where we restrict momenta to the rest frame,

$$p_\mu = p_0 \delta_\mu^0. \quad (66)$$

We label spatial (transverse) directions with latin indices, $i, j \in \{1, 2, 3\}$. It is useful to split the transverse metric variation again into traceless (\tilde{h}_{ij}) and trace part,

$$\bar{h}_{ij} = \tilde{h}_{ij} + \frac{1}{3} \delta_{ij} \bar{h}_{00}, \quad (67)$$

where we have incorporated the tracelessness of $\bar{h}_{\mu\nu}$. The linearised Einstein-Hilbert action (17) gives the following contribution to the kinetic energy

$$\frac{E_{\text{kin}}^{\text{EH}}}{p_0^2} = M_{\text{Planck}}^2 \left(\frac{1}{4} \tilde{h}_{ij}(p)^* \tilde{h}^{ij}(p) - \frac{3}{32} \left| \hat{h}(p) - \frac{4}{3} \bar{h}_0^0(p) \right|^2 \right). \quad (68)$$

We observe that \bar{h}_0^i as well as the combination $\hat{h} + \frac{3}{4} \bar{h}_0^0$ do not contribute. The reason is gauge invariance of the linearised Einstein-Hilbert action. Even with the terms coupling to π there is an invariance under trace (\hat{h}) preserving transformations. For $a \neq 0$ this symmetry is broken by the mass term (as long as we do not transform π). So at least for $a = 0$ we can gauge fix

$$\bar{h}_0^0 = -\frac{4}{3} \hat{h}, \quad \bar{h}_i^0 = 0. \quad (69)$$

Fixing of symmetries involving π transformations will be discussed shortly. We find for the kinetic energy of the π, \hat{h} directions

$$\begin{aligned} \frac{E_{\text{kin}}}{p_0^2} &\sim \left(\hat{h}(p)^*, \pi(p)^* \right) \times \\ &\left(\begin{array}{cc} -\frac{625 M_{\text{Planck}}^2}{864} & -2(a+b)f^2 \\ -2(a+b)f^2 & -\frac{8af^4}{M_{\text{Planck}}^2} (5a+8b) - 4(a+b)p_0^2 \end{array} \right) \begin{pmatrix} \hat{h}(p) \\ \pi(p) \end{pmatrix}. \end{aligned} \quad (70)$$

One of the eigenvalues of the kinetic mixing matrix will change sign at a certain scale. The corresponding eigenvector is identified with the ghost direction. The other direction can be removed by fixing the (additional) gauge symmetry introduced in (60). Now, it is easy to find a value of p_0^2 at which the determinant of the kinetic mixing matrix changes sign. Taking that value to be the ghost mass (m_g) we obtain an estimate

$$m_g^2 \sim \frac{6a^2 f^4}{M_{\text{Planck}}^2 (a+b)} - \frac{16af^4}{M_{\text{Planck}}^2} + \frac{864f^4(a+b)}{625M_{\text{Planck}}^2} + \dots, \quad (71)$$

where we have organised contributions in powers of $a+b$. For $a \neq 0$ the gauge fixing in (69) is not justified. What one could do instead is to include mass terms and modify our definition of the ghost mass such that it corresponds to the scale at which the total energy changes sign. The dots in (71) stand for corrections vanishing for $a = 0$. For the case $a = 0$ we see that the ghost mass is of the same order as the h_μ^μ mass.

B Combining with Extra Dimensions

In this appendix we discuss some aspects of our setup when combined with the idea of extra dimensions, in particular with warped brane worlds. The best known example for such brane worlds are the Randall-Sundrum models [36, 37]. In these models the fine-tuning problem of the cosmological constant is translated into matching conditions arising from delta-function sourced five dimensional Einstein equations [38]. Attempts to turn this into a self-tuning [39, 40] are problematic [41, 42]. Unconventional Lagrangians seem to improve the situation [43–45]. However, problems may arise when taking into account perturbations [46–48] or energy conditions [49].

In this appendix, we discuss a model with unconventional Lagrangian. It provides a Randall-Sundrum II geometry, i.e. a single brane and exponential warping in the bulk. Matching conditions just fix integration constants. However, the effective four dimensional theory is the one discussed in the bulk of the present paper. There, we have seen that perturbations lead to problems. We refrain from investigating perturbations including additional fields in the extra dimensional scenario. We do not expect that they can improve the model.

The action is the sum of a five dimensional bulk term and a 3-brane source

$$S = S_{\text{bulk}} + S_{\text{brane}}. \quad (72)$$

For the bulk action we take a five dimensional version of (1)

$$S = \int d^4x dy \sqrt{-G} \left\{ \frac{R}{2\kappa_5^2} - \lambda_5 \left| \frac{\gamma}{G} \right|^\alpha \right\}, \quad (73)$$

the fifth direction is called y . Metric and curvature are tensors in five dimensions and γ is defined as in (2) with the modification that $M, N \in \{0, 1, 2, 3, 5\}$, η_{MN} is the five dimensional Minkowski metric (again in mostly plus convention) and we used capital G_{MN} for the five dimensional metric. The brane source is described by

$$S_{\text{brane}} = - \int d^4x \sqrt{-G} f(\phi^5) \Big|_{y=0}. \quad (74)$$

(Strictly speaking, G should be replaced by the determinant of the metric induced on the brane. With ansatz (75) there will be no difference.) For f being constant this is just the 4d vacuum energy containing a classical part plus contributions due to vacuum fluctuations of all 4d particle physics fields (which are living on the brane). The ϕ^5 dependence can be included to allow for some non vanishing coupling while preserving four dimensional Lorentz invariance in field space.

For the five dimensional metric we impose a warped ansatz

$$ds^2 = a(y)^2 \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (75)$$

where $\mu, \nu \in \{0, 1, 2, 3\}$. Einstein's equations provide two independent equations (prime on a denotes derivation with respect to y)

$$6 \left(\frac{a'}{a} \right)^2 = -\lambda_5 \kappa_5^2 (1 - 2\alpha) |a|^{-8\alpha} |\gamma|^\alpha, \quad (76)$$

$$3 \frac{a''}{a} + 3 \left(\frac{a'}{a} \right)^2 = -\lambda_5 \kappa_5^2 (1 - 2\alpha) |a|^{-8\alpha} |\gamma|^\alpha - \kappa_5^2 f(\phi^5) \delta(y). \quad (77)$$

The equations for the scalars read

$$2\alpha \lambda_5 \partial_M \left(|a|^{4-8\alpha} |\gamma|^\alpha (\gamma^{-1})^{MN} \partial_N \phi^K \right) = \delta_5^K f'(\phi^5) |a|^4 \delta(y). \quad (78)$$

Our ansatz for the scalars is

$$\partial_M \phi^N = \begin{cases} C \delta_M^N & \text{for } M, N \in \{0, 1, 2, 3\}, \\ \varphi'(y) & \text{for } M = N = 5, \\ 0 & \text{else,} \end{cases} \quad (79)$$

where C is constant. (Continuity of the configurations in the first line would allow to absorb the constant in a redefinition of the first four scalars. It will be useful to keep it, though.) This ansatz solves (78) automatically for $K \in \{0, 1, 2, 3\}$, whereas the fifth equation reads for $y \neq 0$

$$\partial_y \left(\left| \frac{\varphi'}{a^4} \right|^{2(\alpha-1)} \frac{\varphi'}{a^4} \right) = 0. \quad (80)$$

So, for $y \neq 0$ warp factor and fifth scalar are related by

$$\varphi' = c_1 a^4, \quad (81)$$

where c_1 is another integration constant. Plugging this into (76) yields

$$\frac{a'}{a} = \mp A |C^4 c_1|^\alpha, \quad (82)$$

with

$$A = \sqrt{\frac{\kappa_5^2 (2\alpha - 1) \lambda_5}{6}}, \quad (83)$$

and we take the squareroot of a positive real number to be positive. Reality of the solution implies the condition

$$\lambda_5 (2\alpha - 1) > 0. \quad (84)$$

Inserting (82) into (77) yields no new equation for $y \neq 0$. Equation (82) is solved by

$$a = \exp [\mp A |C^4 c_1|^\alpha y], \quad (85)$$

where for $y > 0$ ($y < 0$) we chose the upper (lower) sign to obtain a finite effective Planck mass in four dimensions. Since a has to be continuous a constant factor can be absorbed

into a rescaling of the x^μ . The solution including delta function sources is now obtained by taking solutions for $y > 0$ and $y < 0$ with different integration constants and integrating equations over an infinitesimal interval containing $y = 0$. This procedure yields so called matching conditions relating integration constants at the two sides of the brane. The first four scalar equations imply that C should be the same on both sides of the brane. Einstein equation (77) yields a jump condition on a'

$$a'(+0) - a'(-0) = -\frac{1}{3}a\kappa_5 f|_{y=0}. \quad (86)$$

For integration constants this implies

$$A|C|^{4\alpha}(|c_1^>|^\alpha + |c_1^<|^\alpha) = \frac{1}{3}\kappa_5^2 f|_{y=0}. \quad (87)$$

where superscript $>$ ($<$) characterises the solution for $y > 0$ ($y < 0$). The LHS of (87) is never negative and we conclude that the brane has to have positive tension (as in Randall-Sundrum II). The fifth scalar equation (78) yields another jump condition resulting in

$$|c_1^>|^{2(\alpha-1)} c_1^> - |c_1^<|^{2(\alpha-1)} c_1^< = \frac{|C|^{-8\alpha}}{2\alpha\lambda_5} f'(\phi^5)|_{y=0}. \quad (88)$$

Notice that, up to a subtlety concerning sign, c_1 and C always enter in the same multiplicative combination. So, we should count them as one integration constant. Taking into account the two sides of the brane we still have two integration constants with two matching conditions. So, it seems we have succeeded to obtain a Randall-Sundrum II brane world without fine tuning. They differ, however, in the effective four dimensional theory.

To get an idea about the effective four dimensional theory we first freeze as many moduli as we consistently can, meaning the effective four dimensional theory should be invariant under four dimensional diffeomorphisms with Minkowski space being a solution. It turns out that we have to keep more moduli than just the four dimensional metric as in Randall-Sundrum II. Namely, if we just modified (75) to

$$ds^2 = a(y)^2 g_{\mu\nu}(x) dx^\mu dx^\nu + dy^2, \quad (89)$$

plugged in the solution for the rest and integrated over the fifth direction only the Einstein-Hilbert term and the brane source would give rise to a diffeomorphism invariant theory,

$$S_1 = \int d^4x \sqrt{-g} \left\{ \frac{R}{2\kappa^2} - \Lambda \right\}. \quad (90)$$

Here, the effective gravitational coupling is

$$\frac{1}{\kappa^2} = \frac{|C|^{-4\alpha}}{2A\kappa_5^2} (|c_1^>|^{-\alpha} + |c_1^<|^{-\alpha}), \quad (91)$$

whereas

$$\Lambda = \frac{1}{2} f|_{y=0}. \quad (92)$$

Minkowski space is not a solution which is no surprise since we have not yet taken into account the scalars. To obtain a diffeomorphism invariant theory we have to keep the first four scalars $\phi^\mu(x)$ as moduli. This yields a contribution

$$S_2 = -\lambda \int d^4x \sqrt{-g} \left| \frac{\gamma}{g} \right|^\alpha, \quad (93)$$

where γ is now the determinant of the four dimensional matrix as in (2) and

$$\lambda = \lambda_5 \int dy |a^4|^{1-2\alpha} |\varphi'|^{2\alpha} = \frac{\lambda_5}{4A} |C|^{-4\alpha} (|c_1^>|^\alpha + |c_1^<|^\alpha). \quad (94)$$

Using (87), (83) and (92) this can be rewritten into (6). Although we obtained the Randall-Sundrum geometry the effective four dimensional theory is different. Barring the frozen moduli, it is the same model as discussed in the bulk of the paper. We have seen, linearised gravity cannot be consistently quantised in that model. We do not expect taking into account the frozen moduli will provide an improvement.

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